

**KU LEUVEN**



## Introduction to the Design and Cryptanalysis of Cryptographic Hash Functions

Bart Preneel  
 KU Leuven - COSIC  
 firstname.lastname@esat.kuleuven.be

Sibenik, June 2016



### Hash functions

X.509 Annex D  
 MDC-2  
 MD2, MD4, MD5  
 SHA-1

→

RIPEMD-160  
 SHA-256  
 SHA-512

→

**SHA-3**

*This is an input to a cryptographic hash function. The input is a very long string, that is reduced by the hash function to a string of fixed length. There are additional security conditions: it should be very hard to find an input hashing to a given value (a preimage) or to find two colliding inputs (a collision).*

→ **h** → 1A3FD4128A198FB3CA345932

2

### Applications

- short unique identifier to a string
  - digital signatures
  - data authentication
- one-way function of a string
  - protection of passwords
  - micro-payments
- confirmation of knowledge/commitment
- pseudo-random string generation/key derivation
- entropy extraction
- construction of MAC algorithms, stream ciphers, block ciphers,...

2005: 800 uses of MD5 in Microsoft Windows

3

### Agenda

- Definitions
- Iterations (modes)
- Compression functions
- Constructions
- SHA-3
- Conclusions

4

### Security requirements (n-bit result)

preimage	2 <sup>nd</sup> preimage	collision
?	x ≠ ?	? ≠ ?
↓	↓   ↓	↓   ↓
<b>h</b>	<b>h</b> <b>h</b>	<b>h</b> <b>h</b>
↓	↓   ↓	↓   ↓
h(x)	h(x) = h(x')	h(x) = h(x')
<b>2<sup>n</sup></b>	<b>2<sup>n</sup></b>	<b>2<sup>n/2</sup></b>

5

### Preimage resistance

preimage

?

↓

**h**

↓

h(x)

**2<sup>n</sup>**

- in a password file, one does not store
  - (username, password)
- but
  - (username, hash(password))
- this is sufficient to verify a password
- an attacker with access to the password file has to find a preimage

6

### Second preimage resistance

**2<sup>nd</sup> preimage**

$x \neq ?$

Channel 1: high capacity and insecure

Channel 2: low capacity but secure (= authenticated – cannot be modified)

$h(x) = h(x')$

$2^n$

- an attacker can modify  $x$  but not  $h(x)$
- he can only fool the recipient if he finds a second preimage of  $x$

### Collision resistance

**collision**

$x \neq x'$

$h(x) = h(x')$

$2^{n/2}$

- hacker Alice prepares two versions of a software driver for the O/S company Bob
  - $x$  is correct code
  - $x'$  contains a backdoor that gives Alice access to the machine
- Alice submits  $x$  for inspection to Bob
- if Bob is satisfied, he digitally signs  $h(x)$  with his private key
- Alice now distributes  $x'$  to users of the O/S; these users verify the signature with Bob's public key
- this signature works for  $x$  and for  $x'$ , since  $h(x) = h(x')$

### Pseudo-random function

computationally indistinguishable from a random function

$Adv_n^{prf} = \Pr [K \leftarrow \mathcal{K}: A^{h(\cdot)} \Rightarrow 1] - \Pr [f \leftarrow \mathcal{S} \text{ RAND}(m,n): A^f \Rightarrow 1]$

RAND(m,n): set of all functions from m-bit to n-bit strings

$K \rightarrow h$

$f$

$D$  ? or ?

This concept makes only sense for a function with a secret key

### Brute force (2<sup>nd</sup>) preimage

- multiple target second preimage (1 out of many):**
  - if one can attack  $2^t$  simultaneous targets, the effort to find a single preimage is  $2^{n-t}$
- multiple target second preimage (many out of many):**
  - time-memory trade-off with  $\Theta(2^n)$  precomputation and storage  $\Theta(2^{2n/3})$  time per (2<sup>nd</sup>) preimage:  $\Theta(2^{2n/3})$  [Hellman'80]
- answer: randomize hash function with a parameter S (salt, key, spice,...)**

### Brute force attacks in practice

- (2<sup>nd</sup>) preimage search
  - $n = 128$ : 14 B\$ for 1 year if one can attack  $2^{40}$  targets in parallel
- parallel collision search: small memory using cycle finding algorithms (distinguished points)
  - $n = 128$ : 1 M\$ for 5 hours (or 1 year on 60K PCs)
  - $n = 160$ : 56 M\$ for 1 year
  - need 256-bit result for long term security (30 years or more)

### Quantum computers

- in principle exponential parallelism
- inverting a one-way function:  $2^n$  reduced to  $2^{n/2}$  [Grover'96]
- collision search: can we do better than  $2^{n/2}$ ?
  - $2^{n/3}$  computation + hardware [Brassard-Hoyer-Tapp'98] =  $2^{2n/3}$
  - [Bernstein'09] classical collision search requires  $2^{n/4}$  computation and hardware (= standard cost of  $2^{n/2}$ )

### Properties in practice

- collision resistance is not always necessary
- other properties are needed:
  - PRF: pseudo-randomness if keyed (with secret key)
  - PRO: pseudo-random oracle property
  - near-collision resistance
  - partial preimage resistance (most of input known)
  - multiplication freeness
- how to formalize these requirements and the relation between them?

# Iteration

## (mode of compression function)

### How **not** to construct a hash function

- Divide the message into  $t$  blocks  $x_i$  of  $n$  bits each

### Hash function: iterated structure

- split messages into blocks of fixed length and hash them block by block with a compression function  $f$
- need padding at the end

efficient and elegant... but ...

### Security relation between $f$ and $h$

- iterating  $f$  can degrade its security
  - trivial example: 2<sup>nd</sup> preimage

### Security relation between $f$ and $h$ (2)

- solution: Merkle-Damgård (MD) strengthening
  - fix IV, use unambiguous padding and insert length at the end
- $f$  is collision resistant  $\Rightarrow$   $h$  is collision resistant [Merkle'89-Damgård'89]
- $f$  is ideally 2<sup>nd</sup> preimage resistant  $\Rightarrow$   $h$  is ideally 2<sup>nd</sup> preimage resistant [Lai-Massey'92]
- many other results

### Security relation between f and h (3)

length extension: if one knows  $h(x)$ , easy to compute  $h(x \parallel y)$  without knowing  $x$  or IV

solution: output transformation

### Attacks on MD-type iterations

- long message 2<sup>nd</sup> preimage attack** [Dean-Felten-Hu'99], [Kelsey-Schneier'05]
  - Sec security degrades lineary with number **2<sup>t</sup>** of message **blocks** hashed:  $2^{n-t+1} + t \cdot 2^{n/2+1}$
  - appending the length does not help here!
- multi-collision attack and impact on concatenation** [Joux'04]
- herding attack** [Kelsey-Kohno'06]
  - reduces security of commitment using a hash function from  $2^n$
  - on-line  $2^{n-t} +$  precomputation  $2 \cdot 2^{(n+t)/2} +$  storage  $2^t$

### How (NOT) to strengthen a hash function?

[Coppersmith'85][Joux'04]

- answer: concatenation
- $h_1$  ( $n_1$ -bit result) and  $h_2$  ( $n_2$ -bit result)

intuition: the strength of  $g$  against collision/(2<sup>nd</sup>) preimage attacks is the product of the strength of  $h_1$  and  $h_2$  — if both are “independent”

- but....

### Multiple collisions $\neq$ multi-collision

Assume “ideal” hash function  $h$  with  $n$ -bit result

- $\Theta(2^{n/2})$  evaluations of  $h$  (or steps): 1 collision
  - $h(x)=h(x')$
- $\Theta(r \cdot 2^{n/2})$  steps:  $r^2$  collisions
  - $h(x_1)=h(x'_1)$ ;  $h(x_2)=h(x'_2)$ ; ... ;  $h(x_{r-2})=h(x'_{r-2})$
- $\Theta(2^{2n/3})$  steps: a 3-collision
  - $h(x)=h(x')=h(x'')$
- $\Theta(2^{n(t-1)/t})$  steps: a  $t$ -fold collision (multi-collision)
  - $h(x_1)=h(x_2)=\dots=h(x_t)$

### Multi-collisions on iterated hash function (2)

- for IV: collision for block 1:  $x_1, x'_1$
- for  $H_1$ : collision for block 2:  $x_2, x'_2$
- for  $H_2$ : collision for block 3:  $x_3, x'_3$
- for  $H_3$ : collision for block 4:  $x_4, x'_4$

now  $h(x_1 \parallel x_2 \parallel x_3 \parallel x_4) = h(x'_1 \parallel x_2 \parallel x_3 \parallel x_4) = h(x'_1 \parallel x'_2 \parallel x_3 \parallel x_4) = \dots = h(x'_1 \parallel x'_2 \parallel x'_3 \parallel x'_4)$  **a 16-fold collision (time: 4 collisions)**

### Multi-collisions

[Coppersmith'85][Joux '04]

- finding multi-collisions for an iterated hash function is not much harder than finding a single collision (if the size of the internal memory is  $n$  bits)
- algorithm
  - generate  $R = 2^{n/2}$ -fold multi-collision for  $h_2$
  - in  $R$ : search by brute force for  $h_1$
- Time:  $n \cdot 2^{n/2} + 2^{n/2} \ll 2^{(n+1+n/2)}$

### Multi-collisions

[Coppersmith'85][Joux '04]

consider  $h_1$  ( $n_1$ -bit result) and  $h_2$  ( $n_2$ -bit result), with  $n_1 \geq n_2$ .  
 concatenation of 2 iterated hash functions ( $g(x) = h_1(x) || h_2(x)$ )  
 is **as most as strong as the strongest** of the two (even if both are independent)

- cost of collision attack against  $g$  at most  $n_1 \cdot 2^{n_2/2} + 2^{n_1/2} \ll 2^{(n_1 + n_2)/2}$
- cost of  $(2n)$  preimage attack against  $g$  at most  $n_1 \cdot 2^{n_2/2} + 2^{n_1} + 2^{n_2} \ll 2^{n_1 + n_2}$
- if either of the functions is weak, the attacks may work better

25

### Improving MD iteration

salt + output transformation + counter + wide pipe

security reductions well understood  
 many more results on property preservation  
 impact of theory limited

26

### Improving MD iteration

- degradation with use: salting (family of functions, randomization)
  - or should a salt be part of the input?
- PRO: strong output transformation  $g$ 
  - also solves length extension
- long message  $2^{nd}$  preimage: preclude fix points
  - counter  $f \rightarrow f_i$  [Biham-Dunkelman'07]
- multi-collisions, herding: avoid breakdown at  $2^{n/2}$   
 with larger internal memory: known as wide pipe
  - e.g., extended MD4, RIPEMD, [Lucks'05]

27

### Tree structure: parallelism

[Damgård'89], [Pal-Sarkar'03], [Keccak team'13]

28

### Permutation ( $\pi$ ) based: sponge

if result has  $n$  bits,  $H_1$  has  $r$  bits (rate),  $H_2$  has  $c$  bits (capacity) and the permutation  $\pi$  is "ideal"

collisions	$\min(2^{c/2}, 2^{n/2})$
$2^{nd}$ preimage	$\min(2^{c/2}, 2^n)$
preimage	$\min(2^c, 2^n)$

30

### Modes: summary

- growing theory to reduce security properties of hash function to that of compression function (MD) or permutation (sponge)
  - preservation of large range of properties
  - relation between properties
- it is very nice to assume multiple properties of the compression function  $f$ , but unfortunately it is very hard to verify these
- still no single comprehensive theory

30

# Compression functions

31

### Single block length [Rabin '78]

- Merkle's meet-in-the-middle: ( $2^{\text{nd}}$ ) preimage in time  $2^{n/2}$ 
  - select  $2^{n/2}$  values for  $(x_1, x_2)$  and compute forward  $H_2$
  - select  $2^{n/2}$  values for  $(x_3, x_4)$  and compute backward  $H''_2$
  - by the birthday paradox expect a match and thus a ( $2^{\text{nd}}$ ) preimage

32

### Block cipher ( $E_K$ ) based: single block length

Davies-Meyer

Miyaguchi-Preneel

- output length = block length  $m$ ; rate 1; 1 key schedule per encryption
- 12 secure compression functions (in ideal cipher model)
  - lower bounds: collision  $2^{n/2}$ , ( $2^{\text{nd}}$ ) preimage  $2^n$
- [Preneel+'93], [Black-Rogaway-Shrimpton'02], [Duo-Li'06], [Stam'09],...

33

### Permutation ( $\pi$ ) based

paraoza  
JH

small permutation  
Grøstl

34

### Block cipher ( $E_K$ ) based: double block length ( $3n$ to $2n$ compression)

**Open problems:**

- what is the best collision/preimage security for 2 block cipher calls?
- For optimal collision security: what is the best preimage security for  $s$  block cipher calls? (upper bounds are known)

35

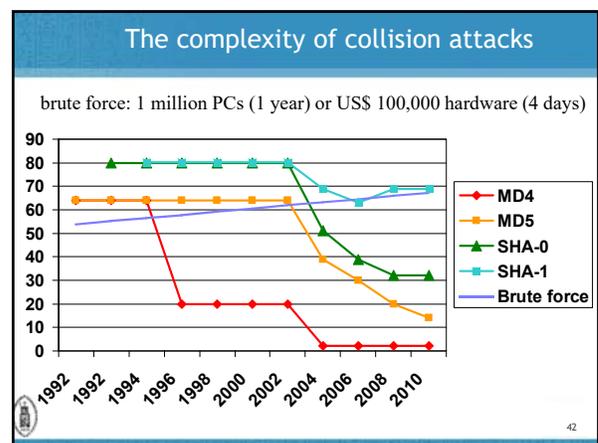
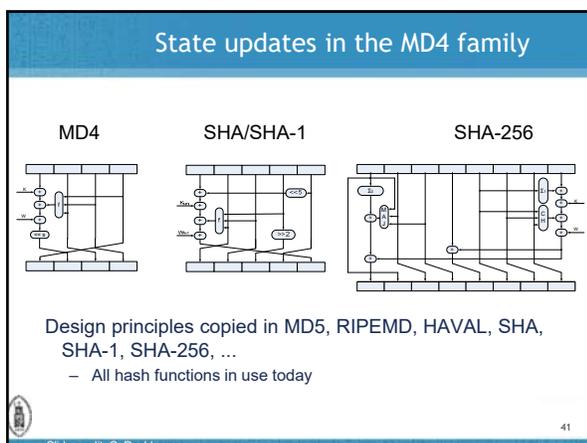
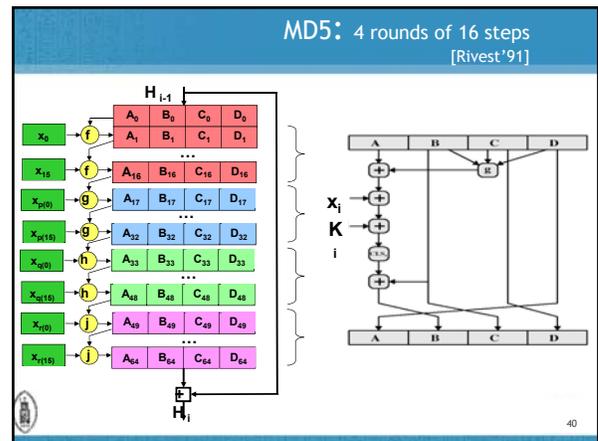
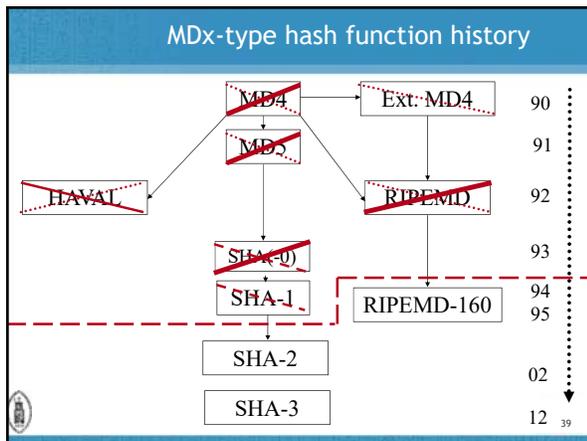
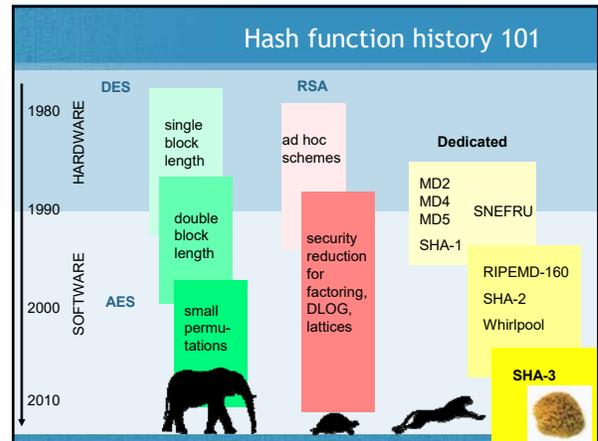
### Iteration modes and compression functions

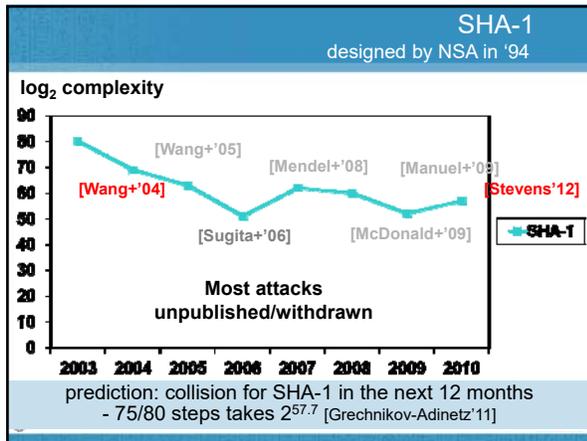
- security of simple modes well understood
- powerful tools available
- analysis of slightly more complex schemes very difficult
- MD versus sponge debate:
  - sponge is simpler
  - sponge easier to extend to authenticated encryption, MAC...
  - should  $x_i$  and  $H_{i-1}$  be treated differently?

36

# Hash function constructions

37





### Rogue CA attack

[Sotirov-Stevens-Appelbaum-Lenstra-Molnar-Osvik-de Weger '08]

- request user cert; by special collision this results in a fake CA cert (need to predict serial number + validity period)

impact: **rogue CA** that can issue certs that are trusted by all browsers

- 6 CAs have issued certificates signed with MD5 in 2008:
  - Rapid SSL, Free SSL (free trial certificates offered by RapidSSL), TC TrustCenter AG, RSA Data Security, Verisign.co.jp

### Collisions for SHA-1 compression function

[Stevens-Karpman-Peyrin'15]

- 10 days on a cluster of 64 GPUs (2K\$)
  - does not lead to a collision for SHA-1 with fixed IV
  - compare to [denBoer-Bosselaers'93] for MD5
- by extrapolation: 100K\$ for SHA-1 collision
- browser industry: planned stop accepting SHA-1 certs in 2017
  - September 2015: 28.2% of certs still use SHA-1

### Upgrades

- RIPMD-160 is good replacement for SHA-1
- upgrading algorithms is always hard
- TLS uses MD5 || SHA-1 to protect algorithm negotiation (up to v1.1)
- upgrading negotiation algorithm is even harder: need to upgrade TLS 1.1 ('06) to TLS 1.2 ('08)**
  - progress in November 2013 (Google, Microsoft)
  - but **TLS 1.2 allows MD5 only!! SLOTH attack [late 2015]**

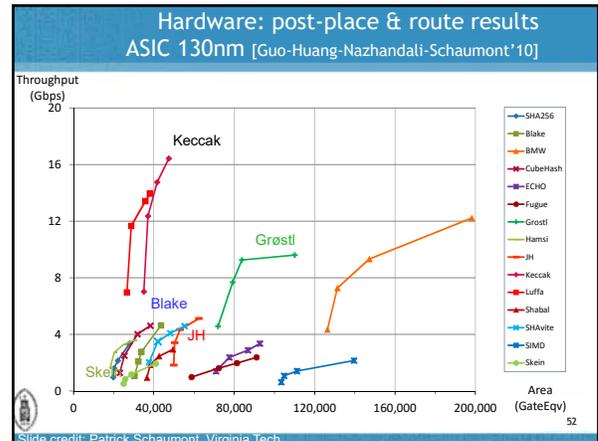
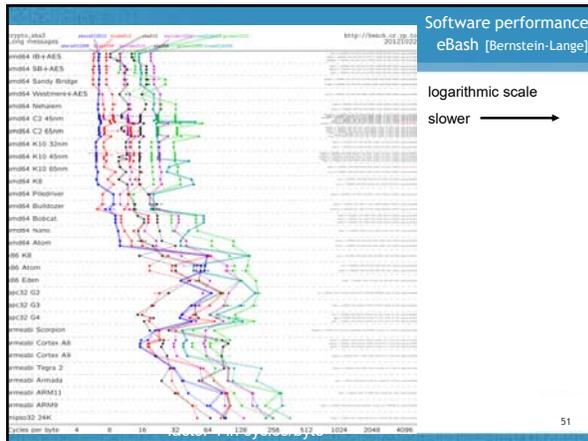
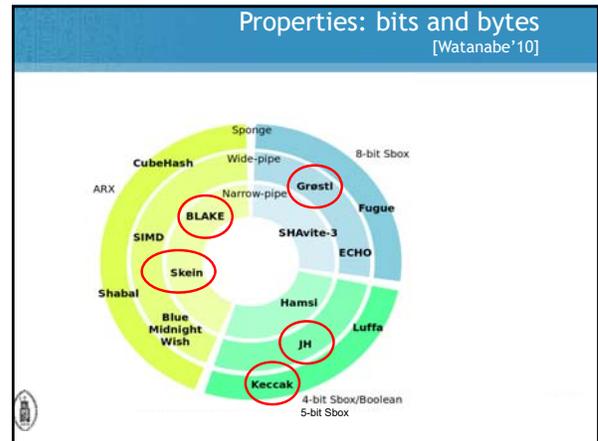
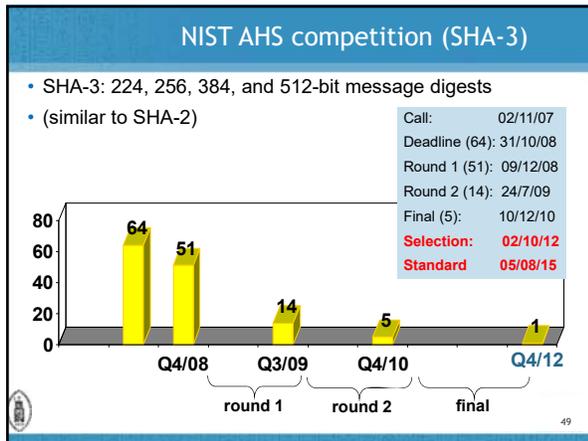
### SHA-2 : FIPS 180

designed by NSA, published in 2002

- SHA-224, SHA-256, SHA-384, SHA-512, SHA-512/256
  - non-linear message expansion
  - 64/80 steps
  - SHA-384 and SHA-512: 64-bit architectures
- SHA-256 collisions: **31/64 steps 2<sup>65.5</sup>** [Mendel+'13]
  - free start collision: 52/64 steps (2<sup>128-c</sup>) [Li+12]
  - non-randomness 47/64 steps (practical) [Biryukov+11][Mendel+11]
- SHA-256 preimages: **45/64 steps (2<sup>256-c</sup>)** [Khovratovitch'12]
- implementations today faster than anticipated
  - 18 cycles/byte on Core 2 (2008) → 7.8 cycles/byte on Haswell (2013)
- adoption accelerated by other attacks on TLS
  - since 2013 deployment in TLS 1.2

# SHA-3

(bits and bytes)



## Keccak

$R = \iota \circ \chi \circ \pi \circ \rho \circ \theta$

permutation: 25, 50, 100, 200, 400, 800, 1600

nominal version:

- 5x5 array of 64 bits
- 24 rounds of 5 steps

[Dinur+13]: collisions 5 out of 24 rounds

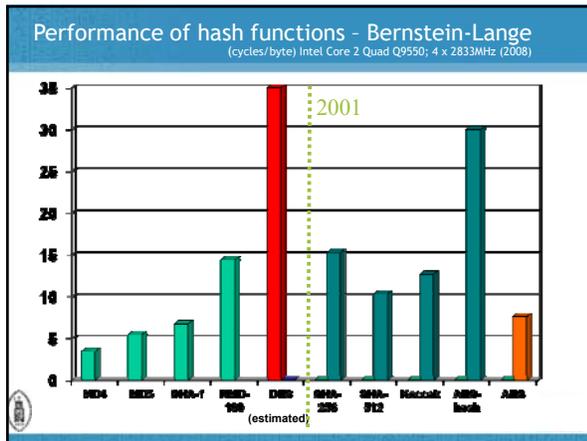
## Keccak: FIPS 202

(published: 5 August 2015)

- append 2 extra bits for domain separation to allow
  - flexible output length (XOFs or eXtensible Output Functions)
  - tree structure (Sakura) allowed by additional encoding
- 6 versions
  - SHA3-224: n=224; c = 448; r = 1152 (72%)
  - SHA3-256: n=256; c = 512; r = 1088 (68%)
  - SHA3-384: n=384; c = 768; r = 832 (52%)
  - SHA3-512: n=512; c = 1024; r = 576 (36%)
  - SHAKE128: n=x; c = 256; r = 1344 (84%)
  - SHAKE256: n=x; c = 512; r = 1088 (68%)

pad 01  
pad 11 for XOF

if result has n bits, H1 has r bits (rate), H2 has c bits (capacity) and the permutation  $\pi$  is "ideal" collisions  $\min(2^{c/2}, 2^{n/2})$   
 2<sup>nd</sup> preimage  $\min(2^{c/2}, 2^n)$   
 preimage  $\min(2^c, 2^n)$



- ### Hash functions: conclusions
- SHA-1 would have needed 128-160 steps instead of 80
  - 2004-2009 attacks: cryptographic meltdown but not dramatic for most applications
  - theory is developing for more robust iteration modes and extra features; still early for building blocks
  - Nirwana: efficient hash functions with security reduction

# The end

Thank you for your attention